SHAFT DESIGN

LECTURE NOTES — MET 4501 — PROF. LEAH GINSBERG

A **SHAFT** IS A ROTATING MEMBER, USUALLY WITH A CIRCULAR CROSS-SECTION, USED TO TRANSMIT POWER OR MOTION.

SOME COMMON SHAFT COMPONENTS ARE:

- GEARS: MECHANICAL COMPONENTS WITH TEETH THAT MESH WITH ANOTHER TOOTHED PART (TYPICALLY ANOTHER GEAR)
- PULLEYS: MECHANICAL COMPONENTS CONSISTING OF A WHEEL WITH A GROOVED RIM THAT IS USED TO CHANGE THE DIRECTION OR MAGNITUDE OF A FORCE. PULLEY WHEELS MATE WITH BELTS, ROPES, OR CABLES.
- BEARINGS: MECHANICAL COMPONENTS DESIGNED TO SUPPORT ROTATING OR MOVING PARTS.

BEST PRACTICES

1. TO MINIMIZE BOTH DEFLECTIONS AND STRESSES, THE SHAFT LENGTH SHOULD BE KEPT AS SHORT AS POSSIBLE AND OVERHANGS MINIMIZED.
2. A CANTILEVER BEAM WILL HAVE A LARGER DEFLECTION THAN A SIMPLY SUPPORTED ONE (STRADDLE MOUNTED) FOR THE SAME LENGTH, LOAD, AND CROSS-SECTION, SO STRADDLE MOUNTING SHOULD BE USED UNLESS A CANTILEVER SHAFT IS DICTATED BY DESIGN CONSTRAINTS.
3. A HOLLOW SHAFT HAS A BETTER STIFFNESS/MASS RATIO (SPECIFIC STIFFNESS) AND HIGHER NATURAL FREQUENCIES THAN A COMPARABLY STRONG OR STIFF SOLID SHAFT, BUT WILL BE MORE EXPENSIVE AND LARGER IN DIAMETER.
4. TRY TO LOCATE STRESS RAISERS AWAY FROM REGIONS OF LARGE BENDING MOMENTS IF POSSIBLE AND MINIMIZE THEIR EFFECTS WITH GENEROUS RADII AND RELIEFS.

5. IF MINIMIZING DEFLECTION IS THE PRIMARY CONCERN, THEN LOW-CARBON STEEL MAY BE THE PREFERRED MATERIAL SINCE ITS STIFFNESS IS AS HIGH AS THAT OF HIGHER STRENGTH STEELS AND A SHAFT DESIGNED FOR LOW DEFLECTION WILL TEND TO HAVE LOW STRESSES.

6. DEFLECTIONS AT GEARS CARRIED ON THE SHAFT SHOULD NOT EXCEED ABOUT 0.005 in AND THE RELATIVE SLOPE BETWEEN GEAR AXES SHOULD BE LESS THAN ABOUT 0.03°.

7. IF PLAIN (SLEEVE) BEARINGS ARE USED, THE SHAFT DEFLECTION ACROSS THE BEARING LENGTH SHOULD BE LESS THAN THE OIL-FILM THICKNESS OF THE BEARING.

8. IF NON-SELF-ALIGNING ROLLING ELEMENT BEARINGS ARE USED, THE SHAFT ANGULAR DEFLECTION SHOULD BE KEPT LESS THAN ABOUT 0.03° AT THE BEARING.

9. IF AXIAL THRUST LOADS ARE PRESENT, THEY SHOULD BE TAKEN TO GROUND THROUGH A SINGLE THRUST BEARING PER LOAD DIRECTION. DO NOT SPLIT AXIAL LOADS BETWEEN THRUST BEARINGS AS THERMAL EXPANSION OF THE SHAFT CAN OVERLOAD THE BEARING.

10. THE FIRST NATURAL FREQUENCY OF THE SHAFT SHOULD BE AT LEAST 3X THE HIGHEST FORCING FREQUENCY EXPECTED IN SERVICE, AND PREFERABLY MUCH MORE. A FACTOR OF 10X OR MORE IS IDEAL, BUT THIS IS OFTEN DIFFICULT TO ACHIEVE IN MECHANICAL SYSTEMS.

CRITICAL SPEEDS OF ROTATING SHAFTS

FREE VIBRATIONS OF AN ELASTIC BODY (I.E. A MACHINE COMPONENT), ALSO CALLED NATURAL VIBRATIONS, OCCUR AT THE NATURAL FREQUENCY.

NATURAL VIBRATIONS ARE DIFFERENT FROM FORCED VIBRATIONS, WHICH HAPPENS AT THE FREQUENCY OF AN APPLIED FORCE (I.E. AT THE ROTATING FREQUENCY OF THE DRIVING MOTOR)

IF THE FORCED FREQUENCY IS EQUAL TO THE NATURAL FREQUENCY OF THE SYSTEM, THE VIBRATIONS' AMPLITUDE INCREASES MANYFOLD. THIS PHENOMENON IS KNOWN AS RESONANCE.

IF WE IDEALIZE OUR SYSTEM AS A SIMPLE MASS-SPRING SYSTEM, THEN THE NATURAL FREQUENCY OF THE SYSTEM CAN BE CALCULATED AS:

$$\omega_n = \sqrt{k/m}$$

WHERE k IS THE SPRING STIFFNESS AND m IS THE MASS OF THE IDEALIZED SYSTEM.

STRATEGIES TO AVOID RESONANCE:

1. KEEP ALL FORCING, OR SELF-EXCITING, FREQUENCIES BELOW THE SYSTEM'S FIRST CRITICAL FREQUENCY.
2. ACCELERATE THE SYSTEM RAPIDLY THROUGH THE RESONANCE, BEFORE THE VIBRATIONS HAVE A CHANCE TO BUILD UP AMPLITUDE.

COUPLING HUBS TO SHAFTS

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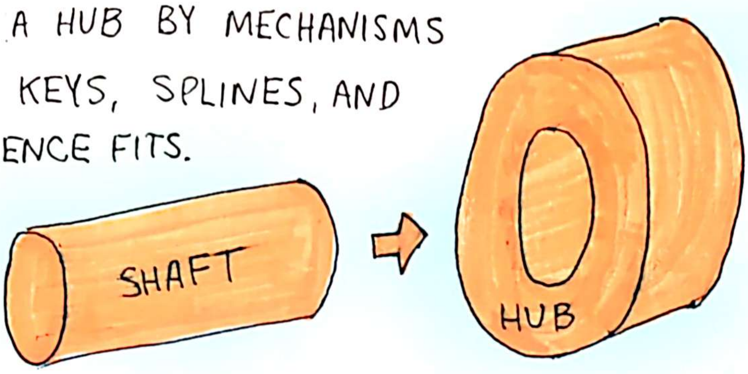
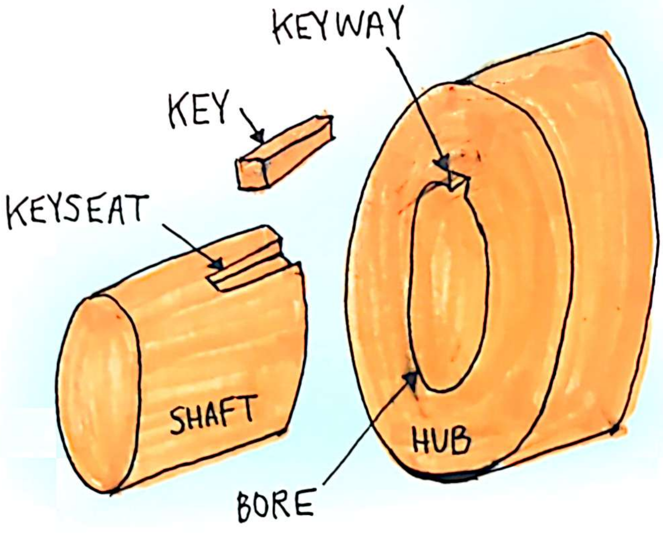
TORQUE IS TRANSMITTED FROM A SHAFT TO A HUB BY MECHANISMS SUCH AS KEYS, SPLINES, AND INTERFERENCE FITS.



A HUB IS A CENTRAL COMPONENT OF A ROTATING ELEMENT, SUCH AS A WHEEL, GEAR, OR PULLEY.

KEYS

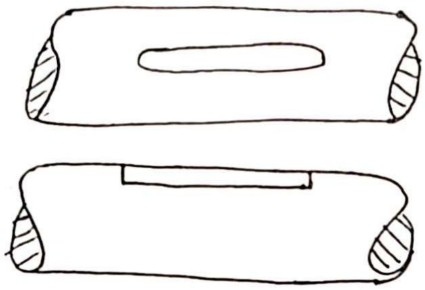


A KEY IS USED TO TRANSMIT TORQUE BETWEEN A SHAFT AND A SHAFT-SUPPORTED ELEMENT.

KEYS ARE AVAILABLE IN SEVERAL STYLES. THE MOST COMMONLY USED KEYS ARE STRAIGHT KEYS, ALSO KNOWN AS PARALLEL KEYS OR MACHINE KEYS.

WHEN DESIGNING A KEYED SHAFT, STRESS CONCENTRATION FACTORS (K_t) SHOULD BE CONSIDERED.

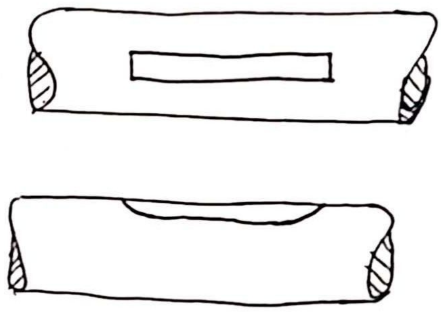
END-MILLED KEYSEATS HAVE SHARP CORNERS AT THE ENDS AND ALONG THE SIDES.



ESTIMATE FOR STRESS-CONCENTRATION FACTOR IN BENDING:

$$K_t = 2.14$$

SLED-RUNNER KEYSEATS HAVE A ROUND TAPER AT THE END(S).



ESTIMATE FOR STRESS-CONCENTRATION FACTOR IN BENDING:

$$K_t = 1.7$$

SEE TABLE 7-1 IN SHIGLEY FOR MORE ESTIMATES OF STRESS-CONCENTRATION FACTORS FOR SHOULDERS AND KEYSEATS.

PARALLEL KEYS MAY BE NEGATIVELY TOLERANCED OR POSITIVELY TOLERANCED:

- WHEN NEGATIVELY TOLERANCED, THE KEY WILL NEVER BE LARGER THAN THE NOMINAL SIZE.
  - ↳ THE KEYSEAT CAN BE CUT WITH A STANDARD MILLING CUTTER AT THE NOMINAL SIZE.
  - ↳ THE KEY WILL FIT IN THE KEYSEAT WITH A SLIGHT CLEARANCE.
- WHEN POSITIVELY TOLERANCED, THE KEY WILL BE SLIGHTLY LARGER THAN THE NOMINAL SIZE.
  - ↳ USED WHEN A CLOSER FIT BETWEEN THE KEY AND KEYSEAT IS DESIRED.
  - ↳ MAY REQUIRE ADDITIONAL MACHINING OF THE KEY.

BACKLASH MAY OCCUR WHEN THE LOADING TORQUE CHANGES DIRECTION.

- ANY CLEARANCE BETWEEN THE KEY AND THE KEYSEAT WILL RESULT IN IMPACT AND HIGH STRESSES.

TO AVOID BACKLASH...

- USE A POSITIVELY TOLERANCED KEY
- USE A SETSCREW TO SECURE A NEGATIVELY TOLERANCED KEY.

KEY CROSS-SECTION IS DICTATED BY THE SHAFT DIAMETER. (FOR EXAMPLE, SEE STANDARD SIZES OF RECTANGULAR KEYS IN SHIGLEY TABLE 7-6.)

KEY LENGTH IS CHOSEN TO CARRY THE TORSIONAL LOAD.

- GENERALLY, SHOULD NOT EXCEED 1.5 TIMES THE SHAFT DIAMETER, TO AVOID EXCESSIVE TWISTING WITH ANGULAR DEFLECTION OF THE SHAFT.
- LIMITED BY THE HUB LENGTH OF THE ATTACHED COMPONENT.

KEYS TYPICALLY FAIL IN TWO WAYS:

	SHEAR FAILURE	BEARING FAILURE
DESCRIPTION	THE KEY IS SHEARED ACROSS ITS WIDTH AT THE INTERFACE BETWEEN THE HUB AND SHAFT.	THE KEY IS CRUSHED DUE TO A COMPRESSIVE LOAD.
CRITICAL STRESS	SHEAR STRESS IS THE SHEAR FORCE DIVIDED BY CUT AREA. $\tau = \frac{F}{A_{shear}} = \frac{T/r}{w \cdot l}$ (↑ FOR A RECTANGULAR KEY)	NORMAL STRESS IS THE CRUSHING FORCE DIVIDED BY THE CRUSHED AREA. $\sigma = \frac{F}{A_{crush}} = \frac{T/r}{\frac{h}{2} \cdot l}$ (↑ FOR A RECTANGULAR KEY)
FACTOR OF SAFETY	COMPARES SHEAR STRESS & MATERIAL'S SHEAR YIELD STRENGTH: $n = \frac{S_{sy}}{\tau} = \frac{0.577 S_y}{\tau}$	COMPARES NORMAL STRESS & MATERIAL'S COMPRESSIVE YIELD STRENGTH: $n = \frac{S_{yc}}{\sigma}$

SPLINES

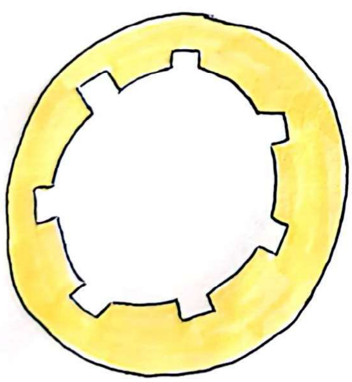
SPLINES ARE FORMED BY CONTOURING THE OUTSIDE OF A SHAFT AND THE INSIDE OF A HUB.

SPLINES HAVE A COUPLE OF ADVANTAGES, COMPARED TO KEYS:

- SPLINES CAN TRANSMIT LARGER LOADS
- SPLINES CAN ACCOMMODATE LARGE AXIAL MOTIONS BETWEEN THE SHAFT AND THE HUB.
- ALL OTHER FACTORS BEING EQUAL, SPLINED CONNECTIONS HAVE A LONGER FATIGUE LIFE THAN KEYED CONNECTIONS BECAUSE THE LOADS ARE MORE EVENLY DISTRIBUTED ALONG THE TEETH
- THE MANUFACTURING COST FOR SPLINED CONNECTIONS IS TYPICALLY HIGHER THAN FOR KEYED CONNECTIONS

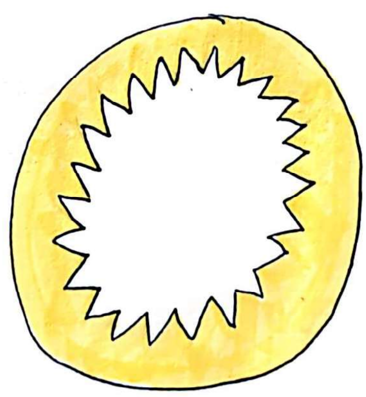
A FEW COMMON SPLINE TYPES ARE SHOWN BELOW:

STRAIGHT SPLINE



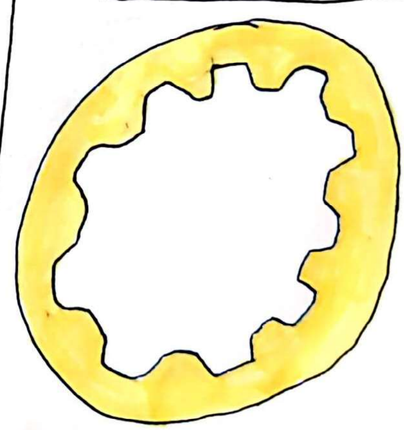
EQUALLY-SPACED, STRAIGHT-SIDED TEETH. LIKE A STRAIGHT KEYED CONNECTION WITH MANY KEYS AROUND THE CIRCUMFERENCE.

SERRATED SPLINE



EQUALLY-SPACED TEETH WITH AN INCLUDED ANGLE. MAINLY USED FOR SMALL DIAMETER DRIVES TO FIT MORE TEETH ON A SMALL CIRCUMFERENCE.

INVOLUTE SPLINE



LIKE STRAIGHT SPLINE, BUT WITH ROUNDED TEETH EDGES. THE SMOOTH TRANSITION BETWEEN TEETH LOWERS STRESS CONCENTRATION, AND INCREASES FATIGUE PROPERTIES.